Extra Credit 17

For equation X^2=X among infinite from left numbers  there are 3 known solution:  0= ...00000, 1= ...00001 and 5^2^infinity  = ...12890625. Give last 8 digits of one more solution whose last digit is 6:  ...??????????6

Let x^2=x and x ends in 6

Let A be the last 8 digits of this number

Therefore, x^2\equiv A\pmod{10^8} and x^2=x\equiv A\pmod{10^8}

Therefore, x^2= x \Rightarrow A^2\equiv A\pmod{10^8}

Hence, we have A^2\equiv A\pmod{10^8}\Rightarrow A(A-1)\equiv 0\pmod{10^8}

Therefore, we must have A(A-1)\equiv 0\pmod{10^k},\,1\leq k\leq 8

Therefore, A(A-1)\equiv 0\pmod{ 100}

We already have A\equiv 6\pmod{10} from the given information:

This means  
A=10k+6\Rightarrow A(A-1)=(10k+6)(10k+5)=10(5k+3)(2k+1)\equiv 0\pmod{100}

That is, (5k+3)(2k+1)\equiv 0\pmod{10}

Testing for a few values, we find that k=7 fits the bill

Therefore, A\equiv 76\pmod{100}

Using this and A(A-1)\equiv 0\pmod{10^{3}} we have

A=100k+76 so that A(A-1)=(100k+76)(100k+75)\equiv 100(25k+19)(4k+3)\equiv 0\pmod{1000}

This means (25k+19)(4k+3)\equiv 0\pmod{10}

Checking for 0\leq k\leq 9 we find that k=3

This means A\equiv 376\pmod{10^3}

Let A=10^3k+376 so that A(A-1)=(10^3k+376)(10^4k+375)=8\times (125k+47)\times 125(8k+3)

That is, A(A-1)=10^3 (125k+47)(8k+3)\equiv 0\pmod{10^4} so that

(125k+47)(8k+3)\equiv 0\pmod{10}

That is, (5k+7)(8k+3)\equiv 0\pmod{10}

Again, checking for single digit k, we have k\equiv -1\pmod{10}\Rightarrow k=9

Hence, A\equiv 9376\pmod{10^4}

Again, let A\equiv 10^4k+9376\Rightarrow A(A-1)=(10^4k+9376)(10^4k+9375)\equiv 0\pmod{10^5}

That is, 10^4\times (625k+586)(16k+15 )\equiv 0\pmod{10^5}

Hence, (625k+586)(16k+15 )\equiv 0\pmod{10}

So that (5k+6)(6k+5 )\equiv 0\pmod{10}

which means k=0

Hence, A\equiv 9376\pmod{10^5} also

Again, let A=10^5k+9376\Rightarrow A(A-1)=(10^5k+9376)(10^5k+9376)=10^5(3125x+293)(32x+3)

That is, 10^5(3125x+293)(32x+3)\equiv 0\pmod{10^6}\Rightarrow (3125x+293)(32x+3)\equiv 0\pmod{10}

Hence, (3125x+293)(32x+3)\equiv 0\pmod{10}\Rightarrow (5x+3)(2x+3)\equiv 0\pmod{10}

Usual checking of single digit 'k' gives k=1

Hence, A\equiv 109376\pmod{10^6}

Let A\equiv 10^6k+109376\Rightarrow A(A-1)=(10^6k+109376)(10^6k+109375)\equiv 0\pmod{10^7}

That is, 10^6(15625k+1709)(64k+7)\equiv 0\pmod{10^7} so that (15625k+1709)(64k+7)\equiv 0\pmod{10}

That is, (5k+9)(4k+7)\equiv 0\pmod{10} which implies after checking, k=7 and soA\equiv 7109376\pmod{10^7}

Finally, let A\equiv 10^7k+7109376\pmod{10^8} so that

A(A-1)\equiv (10^7k+7109376)(10^7k+7109375)\pmod{10^8}

That is, 10^7\times (128k+91)(78125k+55542)\pmod{10^8}

Hence, (128k+91)(78125k+55542)\pmod{10} so that

(8k+1)(5k+2)\pmod{10}

After checking, we have k=8

Hence, A\equiv 87109376\pmod{10^8} so that the last 8 digits are 87109376 as required